LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2024**

PPH3MC01 – QUANTUM MECHANICS I

Date: 07-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**MCQ**
a)	The value of $[x, p_x]$ is a) $\frac{\partial p_x}{\partial x}$ b) $i\hbar \frac{\partial}{\partial x}$ c) $h \frac{\partial}{\partial x}$ d) $p_x \frac{\partial y}{\partial t}$
b)	An electron of mass 9.1 x 10^{-31} kg is moving back and forth between potential barriers 10^{-9} m apart. The energy E_0 for the electron is about __________ a) 6 x 10^{-20} J b) 6 x 10^{-13} J c) 6 x 10^{-7} J d) 6 x 10^{-3} J
c)	The eigenvalues of $\sigma_x, \sigma_y, \sigma_z$ Pauli's spin matrices are_____________. a) 1 b) -1 c) ±1 d) 0
d)	The variation principle is particularly effective when estimating the energy of___________. a) the highest state of any symmetry b) the lowest state of any symmetry c) any state of all symmetry d) none of the above
e)	The method of partial wave analysis is suited only for ______________. a) low energy scattering b) medium energy scattering c) high energy scattering d) none of the above
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	The quantum mechanical operator for the momentum of a particle moving in one dimension is given by _____.
b)	A and B represent two physical characteristics of a quantum system. If A is Hermitian, then for the product AB to be Hermitian, then B should be ___________.
c)	The maximum possible values the magnetic quantum number m_l can take for l = 2 is _____.
d)	___________ is used when the perturbation is not small compared to the unperturbed Hamiltonian.
e)	The optical theorem expression in scattering is ________.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Find $[L_k, r_l]$ and $[L_k, r_k]$ of the Cartesian coordinates $(r_1, r_2\, r_3)$ and Cartesian components of angular momentum (L_1, L_2, L_3) where k,l,m are the cyclic permutations of 1,2,3.
4	What is delta potential? Determine the energy eigenvalue for an attractive one-dimensional delta potential.
5	Express the operators for angular momentum components Lx, Ly, L_Z in spherical polar coordinates.
6	Explain WKB approximation. Obtain the general solution for the one-dimensional Schrodinger equation of a particle moving a region of constant potential V_0.

7	In a scattering experiment, the potential is spherically symmetric and the particles are scattered at such energy that only s and p waves need to be considered. (i) Show that the differential cross section $\sigma(\theta)$ can be written in the form $\sigma(\theta) = a + b\cos\theta + c\, cos^2\, \theta$. What are the values of a, b, c in terms of phase shifts? ii) What is the value of total cross section in terms of a, b, c?
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	a) Show that the commutator $\left[x, [x, H]\right] = -\frac{\hbar^2}{m}$, where H is the Hamiltonian operator. b) A representation is given by the base vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. Construct the transformation matrix U for transformation to another representation consisting of base vectors $\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{i}{\sqrt{2}}\end{pmatrix}$ and $\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-i}{\sqrt{2}}\end{pmatrix}$. Also show that the matrix is unitary. (6+6.5)
9	In the simple harmonic oscillator problem, the creation and annihilation operators are defined as $a^+ = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} x - i\left(\frac{1}{2m\hbar\omega}\right)^{\frac{1}{2}} p$ and $a = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} x + i\left(\frac{1}{2m\hbar\omega}\right)^{\frac{1}{2}} p$. Show that $[a,\ a^+] = 1$; $[a, H] = \hbar\omega a$ and $\langle n\|a^+ a\|n\rangle \geq 0$, where |n> are energy eigenkets of the oscillator.
10	Evaluate the commutators $\left[L_y, L_Z\right], [L^2, L_Z], [L_+, L_-], [L_z, L_-], [L_z, L_+], [L^2, L_+], \left[L_y, L_+\right]$.
11	Define differential scattering cross-section and total cross-section. Obtain the expression for total cross-section and scattering amplitude.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	For a particle trapped in the potential well V(x) = 0 for $\frac{-a}{2} \leq x \leq \frac{a}{2}$ and V(x) = ∞ otherwise, the ground state energy and eigenfunctions are $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$; $\Psi_1 = \sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$. Evaluate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$ and the uncertainty product.
13	a) Verify that $\Psi = A\sin\theta\, \exp(i\emptyset)$, where A is a constant, is an eigen function of $L^2 and\ L_Z$. Find the eigenvalues. b) State Pauli's spin matrices. For Pauli's spin matrices, find (i) $\sigma_x^2 + \sigma_y^2 + \sigma_z^2$ (ii) $\sigma_x\sigma_y + \sigma_y\sigma_x$. (9+6)
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	Setup the Schrodinger wave equation for a square potential barrier with the energy of the particle $E<V_0$. Obtain the expressions for transmissivity and reflectivity and hence explain the alpha particle emission.
15	i) A particle of mass m is confined to move in a potential V(x) = 0 for $0 \leq x \leq a$ and V(x) = ∞ otherwise. The wavefunction of the particle at time t = 0 is $\Psi(x, 0) = A\left(2\, sin\frac{\pi x}{a} + sin\frac{3\pi x}{a}\right)$. Normalize $\Psi(x, 0)$ and find $\Psi(x, t)$. ii) Obtain the matrix form of the rotation operator in three dimensions, when y axis is rotated through an angle θ about the y axis. iii) Find the expectation value of the operator $\hat{A} = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ in the state $\|\Psi> = x\|u> + y\|d>$ where $\|u> = \begin{pmatrix}1\\0\end{pmatrix}$; $\|d> = \begin{pmatrix}0\\1\end{pmatrix}$. (10+5+5)

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